References for seminar on K-convexity

The main goal of the seminar was to prove the inequalities

$$MM^*(B_X) \lesssim K(X) \lesssim \log d_{BM}(X, \ell_2^n) \ . \tag{1}$$

The first inequality is due to Figiel and Tomczak-Jaegermann (1979), using a lemma of Lewis (1979); the second inequality is due to Pisier (1981, Prop. 7). The notion of K-convexity originates with Maurey and Pisier (1976, Rem. 2.10).

This subject is treated in the books of Pisier (1989, Ch. 2, 3) (which uses gaussian functions throughout), of Milman and Schechtman (1986, §§9.8–9.10, 14.6, 15.1–15.4) (which uses Rademacher functions, as we did in seminar), and of Tomczak-Jaegermann (1989, §§12, 14); I also made use of some unpublished notes of Litvak. For asymmetric convex bodies, see Banaszczyk et al. (1999) and Rudelson (2000).

In the proof of the first inequality, our conversion from gaussian averages to Rademacher averages (see Bourgain and Milman, 1987, Lem. 8.6; compare Tomczak-Jaegermann, 1989, eq. 4.2–4.4) cost us a logarithmic factor, which can be avoided in two ways: first, by using gaussian functions throughout (as in Pisier's book); second, by using gaussians for the first inequality and Rademachers for the second, and then showing that the gaussian and Rademacher K-convexity constants are equivalent (see Tomczak-Jaegermann, 1989, Th. 12.5).

The second inequality in (1) is known to be asymptotically sharp due to an example by Bourgain (1984, §5) with $K(X) \gtrsim \log n$. If X is 1-unconditional, then as shown by Pisier (1981, Prop. 8), the second inequality can be strengthened to $K(X) \lesssim \sqrt{\log n}$, which is also asymptotically sharp, since $K(\ell_1^n) \gtrsim \sqrt{\log n}$. The proof of this last fact given in seminar is my own; although the fact itself is well-known, the only published proof I am aware of is that of Banaszczyk et al. (1999, Prop. 5.1), who actually show the stronger result that $MM^*(B_1^n) \gtrsim \sqrt{\log n}$. We deduced as a corollary that ℓ_1 is not K-convex, which was observed already by Maurey and Pisier (1976, Rem. 2.10) from considerations of type and cotype.

My treatment of tensor products of vector spaces was a little idiosyncratic. A standard and thorough treatment is given by Greub (1967, Ch. 1).

For Fourier analysis on finite abelian groups, see the self-contained treatment by Körner (1988, Ch. 103, 104). (Here "self-contained" means "without appeal to the structure theorem of finite abelian groups".)

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